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The $\tau \to \mu\bar{\nu}_i\nu_i$ decay in the Randall–Sundrum background with localized $U(1)_Y$ gauge boson

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Abstract. We study the effects of localization of the $U(1)_Y$ gauge boson around the visible brane and the contributions of the Kaluza–Klein modes of Z bosons on the branching ratio of the lepton flavor violating $\tau \to \mu\bar{\nu}_i\nu_i$ decay. We observe that the branching ratio is sensitive to the amount of localization of the Z boson in the bulk of the Randall–Sundrum background.

1 Introduction

Lepton flavor violating (LFV) interactions are rich from the theoretical point of view, since they exist in the loop level, and the measurable quantities of these decays carry considerable information on the free parameters of the model used. In addition to this, they are clean theoretically, because they are free from nonperturbative effects. The experimental work done stimulates the theoretical studies on LFV decays. The $\mu \to e\gamma$ and $\tau \to \mu\gamma$ processes are among the LFV decays, and the experimental current limits for their branching ratios (BRs) have been predicted as 1.2×10^{-11} [1] and 1.1×10^{-6} [2], respectively.

In the framework of the standard model (SM), which allows for the lepton mixing mechanism, the theoretical values of the BRs of LFV interactions are extremely small. Therefore, one needs to search new models beyond the SM to enhance these numerical values, and the two Higgs doublet model (2HDM), with flavor changing neural currents (FCNC) at tree level, is one of the candidates. In this model, the LFV interactions are induced by the internal neutral Higgs bosons h^0 and A^0 with the help of the Yukawa couplings, appearing as free parameters, which can be determined by the experimental data.

The present work is devoted to the analysis of $\tau \to \mu\bar{\nu}_i\nu_i$, $i = e, \mu, \tau$, decay in the 2HDM with the inclusion of a single extra dimension, respecting the Randall–Sundrum model (see [3] and [4] for the calculation of the BR of the same decay in the 2HDM without and with flat extra dimensions). Here, the LFV transition $\tau \to \mu$ is driven by the internal scalar bosons h^0 and A^0 and the internal Z boson connects this transition to the $\bar{\nu}\nu$ output (see Fig. 1).[1]

The hierarchy problem between weak and Planck scales could be explained by introducing the extra dimensions. The model introduced by Randall and Sundrum [5, 6] (the RS1 model) is related to the non-factorizable geometry where the gravity is localized on a 4D brane, the so-called Planck (hidden) brane, which is away from another 4D brane, which is the visible (TeV) one where we live. In this scenario, the hierarchy is generated by the warp factor, which is an exponential function of the compactified radius in the extra dimension. There are other scenarios based on the RS1 background in the literature [7–16]. In [8, 9] the behavior of the U(1) gauge boson, accessible to the extra dimension in the RS1 background, has been studied, and it was observed that the massless mode was not localized in the extra dimension. Furthermore, the result was obtained that the KK excitations have large couplings to the boundaries and this was not a phenomenologically favorable scenario since, for a perturbative theory, it would have been necessary to push the visible scale to energies greater than TeV. To have a zero mode localized in the bulk and to get small couplings of the KK modes with the boundaries, the idea of brane localized mass terms has been considered for scalar fields [10]. These terms could change the boundary conditions to get a zero mode localized solution. In [10, 11], the bulk fermions were considered in the RS1 background. Reference [12] was devoted to extensive work on the bulk fields in various multi-brane models. In [16], the U(1) gauge field with bulk and boundary mass terms were taken into account and only the $U(1)_Y$ gauge field was considered as localized in the bulk.

In this work, we study the effects of localization of the $U(1)_Y$ gauge boson around the visible brane and analyze the contributions of the KK modes of Z bosons on the BRs of the LFV $\tau \to \mu\bar{\nu}_i\nu_i$ decay, by following the idea that the

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[1] Here, we respect the assumption that the Cabibbo–Kobayashi–Maskawa (CKM) type matrix in the leptonic sector does not exist and the charged flavor changing (FC) interactions vanish.

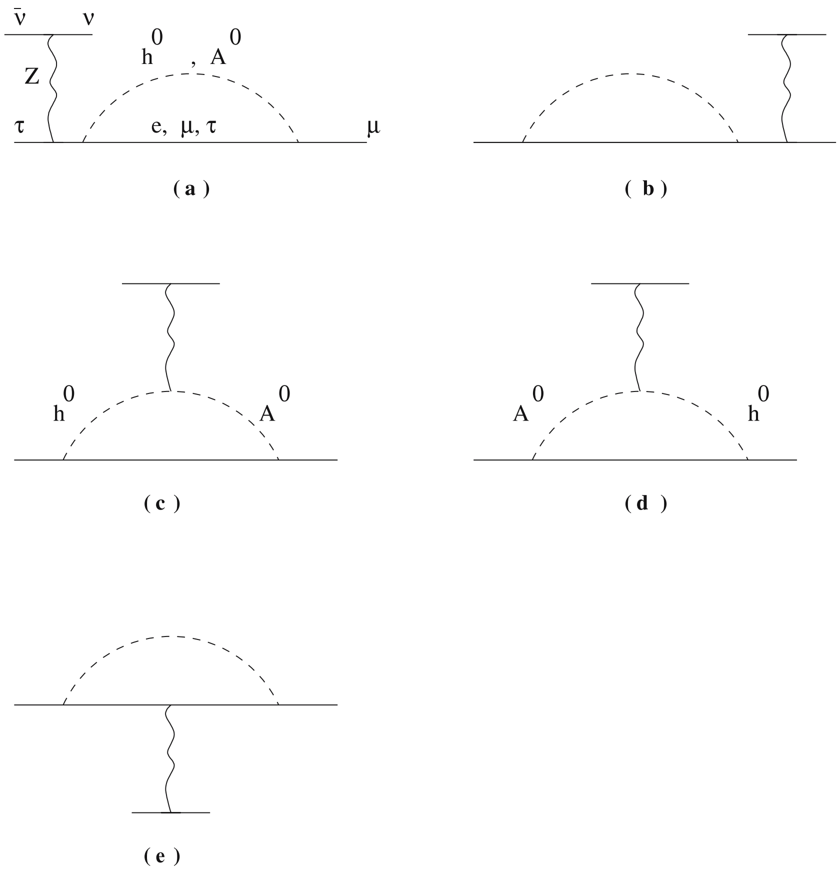

Fig. 1. One loop diagrams contribute to $\tau \to \mu\bar{\nu}_i\nu_i$, $i = e, \mu, \tau$, decay due to the neutral Higgs bosons h^0 and A^0 in the 2HDM. Solid lines represent leptons and neutrinos, *curly* (*dashed*) *lines* represent the virtual Z boson and its KK modes (h^0 and A^0 fields)

$\mathrm{U}(1)_Y$ gauge field is accessible to the extra dimension in the RS1 background and the other particles, including the ones in the new Higgs doublet lie on the visible brane.

The paper is organized as follows: In Sect. 2, we present the theoretical expression for the decay width of the LFV decay $\tau \to \mu\bar{\nu}_i\nu_i$, $i = e, \mu, \tau$, in the framework of the 2HDM by considering the non-zero localization of the $\mathrm{U}(1)_Y$ gauge in the RS1 background. Section 3 is devoted to discussion and our conclusions. In Appendix A, we present the construction of the model. Appendix B is devoted to the explicit expressions of the functions appearing in the general effective vertex for the interaction of the off-shell Z boson with a fermionic current.

2 The effect of the localized $\mathrm{U}(1)_Y$ gauge boson on the $\tau \to \mu\bar{\nu}_i\nu_i$ decay in the Randall–Sundrum background

LFV $\tau \to \mu\bar{\nu}_i\nu_i$ decay exists at least in the one loop level and, therefore, the physical quantities, like the BR, contain rich information on the model used and the free parameters existing. In the 2HDM with the FCNCs at tree level, the LFV interactions exist with larger BRs, compared to the ones obtained in the extended SM with massive neutrinos. These decays are driven by the Yukawa interaction Lagrangian and the strength of the interaction is controlled by the new Yukawa couplings. The additional effects coming from the possible extra dimension(s) brings about new contributions to the BRs of these decays and, in the present work, we study these effects by assuming that the $\mathrm{U}(1)_Y$ gauge boson is accessible to the extra dimension and that it is localized on the visible brane, in the RS1 background.

The RS1 model is a higher dimensional scenario that is based on a non-factorizable geometry and the hierarchy is generated by the warp factor. This model is based on the idea that the gravity is localized on the so called hidden brane, which is one of the boundaries of the S^1/Z_2 orbifold extra dimension, and extended into the bulk with varying strength; however, the SM fields live on another brane, the so called visible brane, which is the other boundary of the extra dimension. In this scenario, the 5D cosmological constant is non-vanishing and both branes have equal and opposite tensions so that the low energy effective theory has flat 4D spacetime. The metric of such a 5D world reads

$$\mathrm{d}s^2 = \mathrm{e}^{-2ky}\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + \mathrm{d}y^2\,, \tag{1}$$

where k is the bulk curvature constant, R is the compactification radius, $y = R|\theta|$, $0 \le |\theta| \le \pi$ and $\mathrm{e}^{-kR|\theta|}$ is the warp factor, which causes all the mass terms to be rescaled on the visible brane for $\theta = \pi$. With a rough estimate, $kR \sim 11$–12, all mass terms are brought down to the TeV scale.

The LFV $\tau \to \mu\bar{\nu}_i\nu_i$ decay is induced by the $\tau \to \mu Z^*$ transition and the $Z^* \to \bar{\nu}_i\nu_i$ process (see Fig. 1). The

$\tau \to \mu Z^*$ transition, which needs the FCNC at tree level, is driven by the internal new neutral Higgs bosons h^0 and A^0, which are living on the visible brane. Now, we present the Yukawa interaction, which is responsible for the $\tau \to \mu$ transition of the decay,

$$\mathcal{S}_Y = \int \mathrm{d}^5x\sqrt{-g}\left(\xi^E_{ij}\bar{l}_{iL}\phi_2 E_{jR} + \mathrm{h.c.}\right)\delta(y-\pi R)\,, \qquad (2)$$

where L and R denote the chiral projections $L(R) = 1/2 \times (1 \mp \gamma_5)$, ϕ_2 is the new scalar doublet, l_{iL} (E_{jR}) are lepton doublets (singlets), ξ^E_{ij},[2] with family indices i, j, are the Yukawa couplings, which induce the FV interactions in the leptonic sector, and g is the determinant of the metric (see (1)).[3] We assume that the Higgs doublet ϕ_1 has a non-zero vacuum expectation value to ensure the ordinary masses of the gauge fields and the fermions; however, the second doublet has no vacuum expectation value, namely, we choose the doublets ϕ_1 and ϕ_2 and their vacuum expectation values as

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ \mathrm{i}\chi^0 \end{pmatrix}\right];$$
$$\phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H^+ \\ H_1 + \mathrm{i}H_2 \end{pmatrix}, \qquad (3)$$

and

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}; \quad \langle\phi_2\rangle = 0\,. \qquad (4)$$

This choice ensures that the mixing between the neutral scalar Higgs bosons is switched off and it would be possible to separate the particle spectrum so that the SM particles are collected in the first doublet and the new particles in the second one.[4]

In our analysis, we further assume that the $\mathrm{U}(1)_Y$ gauge boson is accessible to the extra dimension and the other particles, including the $\mathrm{SU}(2)_L$ gauge bosons, confined on the visible brane. This leads to an effective localization of the Z boson in the bulk of the RS1 background. Therefore, the physical quantities related to the decay studied get additional contributions coming from the internal Z boson and its KK modes (see [16] for details and Appendix A for a summary of the construction of the model).

Now, we present the general effective vertex for the interaction of the off-shell Z boson with a fermionic current

$$\Gamma^{(\mathrm{REN})}_\mu(\tau \to \mu Z^*) = f_1\gamma_\mu + f_2\gamma_\mu\gamma_5 + f_3\sigma_{\mu\nu}k^\nu + f_4\sigma_{\mu\nu}\gamma_5 k^\nu\,, \qquad (5)$$

where k is the momentum transfer, $k^2 = (p-p')^2$, p (p') is the four momentum vector of the incoming (outgoing) lepton and the explicit expressions for the functions f_1, f_2, f_3 and f_4 are given in Appendix B. The matrix element M of the $\tau \to \mu\bar{\nu}_i\nu_i$ process is obtained by connecting the $\tau \to \mu$ transition to the $\bar{\nu}_i\nu_i$ pair with the internal Z boson and its KK modes (see Fig. 1). For the nth internal Z KK mode contribution, the coupling g in f_i, $i = 1,\ldots,4$, and at the $Z^{(n)}\bar{\nu}_i\nu_i$ vertex is replaced by $g^{(n)} \sim \frac{g}{\sqrt{\alpha}}$ (see (A.9) for the parameter α), and the Z boson KK n mode propagator is obtained by using the KK mode mass m_n (see (A.11)). Using the matrix element M, the decay width Γ of the decay under consideration can be obtained in the τ lepton rest frame with the help of the well known expression

$$\mathrm{d}\Gamma = \frac{(2\pi)^4}{2m_\tau}|M|^2\delta^4\left(p - \sum_{i=1}^{3} p_i\right)\prod_{i=1}^{3}\frac{\mathrm{d}^3p_i}{(2\pi)^3 2E_i}\,, \qquad (6)$$

where p (p_i, $i = 1,2,3$) is the four momentum vector of the τ lepton (μ lepton, incoming ν, outgoing ν).

3 Discussion

The BRs of the LFV decays are negligible in the SM, with non-zero neutrino masses. The extension of the Higgs sector brings about new LFV vertices with the help of the new Higgs scalars. 2HDM with FCNC at tree level is the most primitive model that leads to the LFV interactions and the $\tau \to \mu\bar{\nu}_i\nu_i$ is induced by the LFV $\tau \to \mu$ transition that exists at least in the one loop level, in this model. The Yukawa couplings $\bar{\xi}^E_{\mathrm{N},ij}$, $i,j = e,\mu,\tau$, are essential parameters driving the lepton flavor violation and they should be fixed by present and forthcoming experiments.[5] In our calculations, we assume that the Yukawa couplings $\bar{\xi}^E_{\mathrm{N},ij}$ are symmetric with respect to the indices i and j and take $\bar{\xi}^E_{\mathrm{N},ij}$, $i,j = e,\mu$, smaller compared to $\bar{\xi}^E_{\mathrm{N},\tau i}$, $i = e,\mu,\tau$, since we consider the strengths of these couplings to be related with the masses of the leptons denoted by their indices. Therefore, we take only the τ lepton as the internal one and we choose the couplings $\bar{\xi}^E_{\mathrm{N},\tau\tau}$ and $\bar{\xi}^E_{\mathrm{N},\tau\mu}$ as non-zero. The upper limit of the coupling $\bar{\xi}^E_{\mathrm{N},\tau\mu}$ has been estimated as 30 GeV (see [17] and references therein) by assuming that the new physics effects cannot exceed the experimental uncertainty 10^{-9} in the measurement of the muon anomalous magnetic moment. In our numerical calculation we choose $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 1$ GeV by respecting this upper limit. Since there is no restriction for the Yukawa coupling $\bar{\xi}^E_{\mathrm{N},\tau\tau}$, the numerical values we use are greater than $\bar{\xi}^E_{\mathrm{N},\tau\mu}$.

The addition of a single extra spatial dimension brings about a new contribution to the BR of the decay under consideration and the source of this contribution is the KK excitations of the particles, which live in the bulk. Here, we study the BR of the LFV process $\tau \to \mu\bar{\nu}_i\nu_i$ in the framework of the 2HDM, including the extra dimension effects in the RS1 scenario. The RS1 model is an alternative sce-

[2] In the following, we replace ξ^E with ξ^E_{N} where "N" denotes the word "neutral".

[3] Notice that the term $\sqrt{-g} = \mathrm{e}^{-4ky}$ is embedded into the redefinitions of the fields on the visible brane for $y = \pi R$, namely, they are warped as $\phi_2 \to \mathrm{e}^{k\pi R}\phi_2^{\mathrm{warp}}$, $l_i \to \mathrm{e}^{3k\pi R/2}l_i^{\mathrm{warp}}$ and in the following we use warped fields without the warp index.

[4] Here H_1 (H_2) is the well known mass eigenstate h^0 (A^0).

[5] The dimensionful Yukawa couplings $\bar{\xi}^E_{\mathrm{N},ij}$ are defined as $\xi^E_{\mathrm{N},ij} = \sqrt{\frac{4G_{\mathrm{F}}}{\sqrt{2}}}\bar{\xi}^E_{\mathrm{N},ij}$.

nario to solve the well known hierarchy problem. It is based on the assumption that the extra dimension is compactified into an S^1/Z_2 orbifold with two 4D surfaces (branes) at the boundaries in a 5D world and the extra dimensional bulk is populated by gravity, which is localized on the hidden brane. Furthermore, the SM particles live on the so called visible brane. However, in our case, we follow the idea [16] that the $U(1)_Y$ gauge field is accessible to the extra dimension in the RS1 background, and the other particles, including the new Higgs doublet, lie on the visible brane.[6] With the help of the boundary mass term (see (A.6)) it is possible to get the zero mode term and this mode is localized around the visible brane with the special choice of the parameters, a and α, related to the bulk and boundary mass terms (A.8). If the parameter α vanishes, namely, there is no boundary mass term, the KK mode coupling to the fermions becomes almost one order larger compared to the zero mode one and, in order to obtain a perturbative theory, it would be necessary to push the visible scale to energies greater than TeV (see the discussion given in [8, 9]). This is not a phenomenologically favorable scenario. For $\alpha > 0$ the zero mode is localized around the visible brane and the KK mode fermion coupling is small enough for $\alpha > 1$ to obey the phenomenology. Notice that one gets the original RS1 model for infinitely large α.

In the present work, we study the effects of localization of the $U(1)_Y$ gauge boson and the contributions of the KK modes of the Z boson on the BRs of the LFV $\tau \to \mu \bar{\nu}_i \nu_i$ decay. Throughout our calculations we use the input values given in Table 1.

In Fig. 2, we present the dependence on the parameter a of the BR of the LFV $\tau \to \mu \bar{\nu}_i \nu_i$ decay for $\bar{\xi}^E_{N,\tau\tau} = 10$ GeV and $\bar{\xi}^E_{N,\tau\mu} = 1$ GeV. The solid (dashed; small dashed; dotted) line represents the BR without extra dimension (with extra dimension for $kR = 12$, $k = 10^{18}$ GeV; $kR = 11$, $k = 10^{18}$ GeV; $kR = 11$, $k = 10^{17}$ GeV). This figure shows that the BR is of the order of the magnitude of 10^{-6} for the free parameters used and it decreases with increasing values of a. This is due to the fact that the interaction coupling of the Z boson KK modes to the fermions becomes weak with the increasing values of the zero mode localization parameter α and the increase in KK mode masses results in a suppression in the KK mode contribution. For the limit $a \to \infty$, this coupling vanishes, the zero mode is confined on the visible brane and we obtain the numerical values of the BR for the original RS1 model, where all 2HDM particles are confined to the visible brane. On the other hand, we observe that for decreasing values of kR the BR becomes smaller, because of the weaker coupling $g^{(n)}$.

Figure 3 is devoted to the dependence on the parameter k of BR($\tau \to \mu \bar{\nu}_i \nu_i$) for $\bar{\xi}^E_{N,\tau\tau} = 10$ GeV, $\bar{\xi}^E_{N,\tau\mu} = 1$ GeV and $kR = 11$. The solid (dashed; small dashed; dotted) line represents the BR without extra dimension (with an extra dimension for $a = 0.1$; 0.5; 1.0). It is observed that the BR decreases with increasing values of k. Here the KK mode mass increases with k for fixed kR and, as a result, the BR is suppressed and it reaches the one that is obtained without the extra dimension.

Finally, for completeness, we study the dependence on the Yukawa coupling of the BR for different values of the parameter a. Figure 4 represents the $\bar{\xi}^E_{N,\tau\tau}$ depen-

Table 1. The values of the input parameters used in the numerical calculations

Parameter	Value
m_μ	0.106 GeV
m_τ	1.78 GeV
m_{h^0}	100 GeV
m_{A^0}	200 GeV
G_F	1.16637×10^{-5} GeV^{-2}

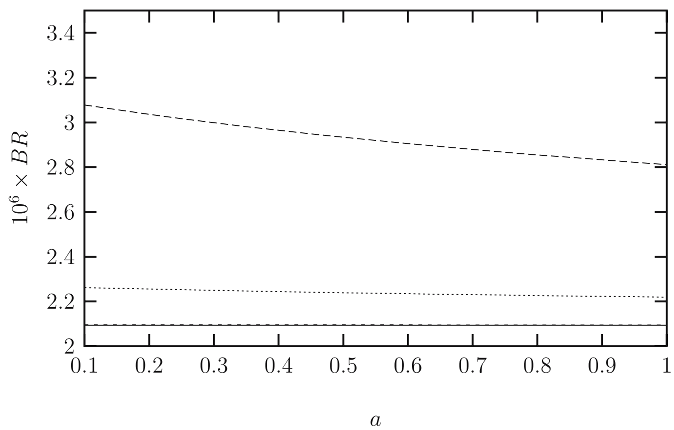

Fig. 2. The parameter a dependence of the BR of the LFV $\tau \to \mu \bar{\nu}_i \nu_i$ decay for $\bar{\xi}^E_{N,\tau\tau} = 10$ GeV and $\bar{\xi}^E_{N,\tau\mu} = 1$ GeV. The *solid* (*dashed*; *small dashed*; *dotted*) *line* represents the BR without extra dimension (with an extra dimension for $kR = 12$, $k = 10^{18}$ GeV; $kR = 11$, $k = 10^{18}$ GeV; $kR = 11$, $k = 10^{17}$ GeV)

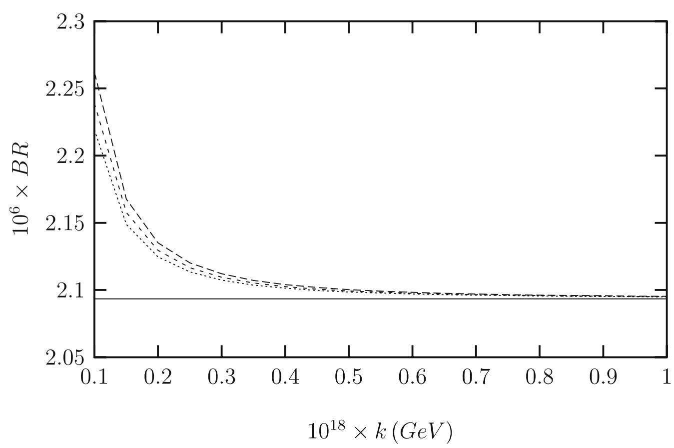

Fig. 3. The dependence on the parameter k of the BR of the LFV $\tau \to \mu \bar{\nu}_i \nu_i$ decay for $\bar{\xi}^E_{N,\tau\tau} = 10$ GeV, $\bar{\xi}^E_{N,\tau\mu} = 1$ GeV and $kR = 11$. The *solid* (*dashed*; *small dashed*; *dotted*) *line* represents the BR without extra dimension (with an extra dimension for $a = 0.1$; 0.5; 1.0)

[6] Here, we assume that the corresponding gauge field makes a small contribution to the bulk energy density and it does not disturb the solution of the Einstein equations.

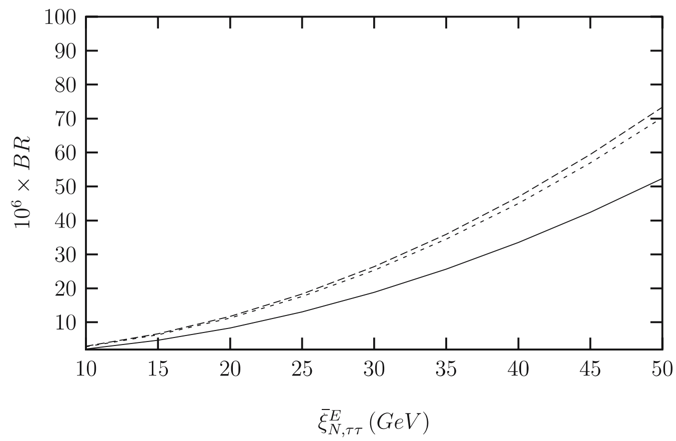

Fig. 4. Dependence on $\bar{\xi}^E_{\mathrm{N},\tau\tau}$ of the BR of the LFV $\tau \to \mu\bar{\nu}_i\nu_i$ decay for $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 1$ GeV, $kR = 12$ and $k = 10^{18}$ GeV. The *solid* (*dashed*; *small dashed*) *line* represents the BR without extra dimension (with an extra dimension for $a = 0.5$; 1.0)

dence of the BR of the LFV $\tau \to \mu\bar{\nu}_i\nu_i$ decay, for $\bar{\xi}^E_{\mathrm{N},\tau\mu} = 1$ GeV, $kR = 12$ and $k = 10^{18}$ GeV. The solid (dashed; small dashed) line represents the BR without extra dimension (with an extra dimension for $a = 0.5$; 1.0). The BR is sensitive to the Yukawa couplings, as it should, and the increase in the parameter a pushes the numerical value of the BR to the one obtained without extra dimension because of the strong localization of the Z field around the visible brane. This is the case that the original RS1 model is reached, where the extra dimension effects are switched off for the process under consideration.

As a result, the BR of the LFV $\tau \to \mu\bar{\nu}_i\nu_i$ decay is sensitive to the parameter a, and it decreases with increasing values of a. In addition to this, the BR decreases with increasing values of k for fixed kR, since the masses of the KK modes are proportional to k. The sensitivity of $\mathrm{BR}(\tau \to \mu\bar{\nu}_i\nu_i)$ to the extra dimension effects in RS1 model is informative; therefore, the more accurate future experimental measurement of this decay may ensure a possible test for the existence of the model used and the determination of signals coming from the extra dimensions.

Appendix A: Construction of the model

Now, we would like to summarize the construction of the model [16], by respecting the assumption that the $U(1)_Y$ gauge boson is accessible to the extra dimension and the $SU(2)_L$ gauge bosons and the other particles, fermions, Higgs bosons, are living on the visible brane, in the RS1 background. The starting point is the 5D action

$$S = \int \mathrm{d}^5x\sqrt{-g}\Big\{-\frac{1}{4}F^{MN}F_{MN} + \left(-\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} + (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)\right)\delta(y-\pi R)\Big\}, \tag{A.1}$$

with the field strength tensors F^{MN} and $G^{a\mu\nu}$ for the $U(1)_Y$ and $SU(2)_L$ gauge bosons, where $M, N = 0, \ldots, 4$; $\mu, \nu = 0, \ldots, 3$. The covariant derivative D_μ reads

$$D_\mu = \partial_\mu - \frac{\mathrm{i}}{2}gA^a_\mu(x)\sigma^a - \mathrm{i}g_5YB_\mu(x,y)\,, \tag{A.2}$$

where $B_\mu(x,y)$, $A^a_\mu(x)$ are the $U(1)_Y$ and $SU(2)_L$ gauge bosons, respectively; g and g_5 are the corresponding couplings to the Higgs boson and σ^a ($Y = 1/2$) is the Pauli spin matrix (the hypercharge). Here the gauge field $B_\mu(x,y)$ is expanded to its Kaluza–Klein (KK) modes by

$$B_\mu(x,y) = \sum_n B^{(n)}_\mu(x)\chi^{(n)}(y)\,, \tag{A.3}$$

and $\chi^{(n)}(y)$ satisfy the differential equation [8,9]

$$\left(\frac{\partial^2}{\partial y^2} - 2k\frac{\partial}{\partial y} - m^2 + \mathrm{e}^{2ky}m_n^2\right)\chi^{(n)}(y) = 0\,, \tag{A.4}$$

with the normalization condition

$$\int_0^{\pi R}\mathrm{d}y\chi^{(n)}(y)\chi^{(m)}(y) = \delta_{nm}\,. \tag{A.5}$$

Here k is the curvature scale and m is the bulk mass of the gauge field $B_\mu(x,y)$, which is of the order of the magnitude of k. To obtain the zero mode, which appears in the construction of the photon and the Z boson fields, the boundary mass term

$$S' = -\int \mathrm{d}^5x\sqrt{-g}\alpha kB^\mu(x,y)B_\mu(x,y)(\delta(y) - \delta(y-\pi R)) \tag{A.6}$$

is considered[7] and this term induces the boundary condition

$$\left(\frac{\partial B_\mu(x,y)}{\partial y} - \alpha kB_\mu(x,y)\right)\Bigg|_{y=0,\pi R} = 0\,, \tag{A.7}$$

which results in a non-vanishing zero mode with fine tuning of the parameters α and $a = m^2/k^2$,

$$\alpha_\pm = 1 \pm \sqrt{1+a}\,. \tag{A.8}$$

Finally, the normalized zero mode is obtained as

$$\chi^{(0)} = \sqrt{\frac{2\alpha k}{\mathrm{e}^{2\alpha k\pi R} - 1}}\mathrm{e}^{\alpha ky}\,, \tag{A.9}$$

and the n mode reads

$$\chi^{(n)}(y) = N_n\mathrm{e}^{ky}\left(J_\nu\left(\frac{m_n}{k}\mathrm{e}^{ky}\right) + b_{n\nu}Y_\nu\left(\frac{m_n}{k}\mathrm{e}^{ky}\right)\right), \tag{A.10}$$

with the normalization constant N_n, and the parameter $\nu = \sqrt{1+a}$. Using the boundary conditions at $y = 0$ and

[7] The idea of brane localized mass terms has been considered for scalar fields in [7, 10].

$y=\pi R$ (see (A.7)), the mass spectrum of the KK modes ($n=1,2,\dots$) reads

$$m_n \simeq \left(n \pm \frac{1}{2}\alpha_\pm - \frac{1}{4}\right)\pi k \mathrm{e}^{-k\pi R}\,, \tag{A.11}$$

for $k\mathrm{e}^{-k\pi R} \ll m_n \ll k$.

Now, we present the mass Lagrangian for the photon and Z fields. After allowing for the Higgs boson vacuum expectation value (see (4)) and expanding the U(1)$_Y$ gauge field $B_\mu(x,y)$ to the KK modes (see (A.3)), the mass Lagrangian becomes

$$\begin{aligned} L_\mathrm{m} = & \sum_{n=1}^{\infty} \frac{1}{2} m_n^2 \left(B_\mu^{(n)}(x)\right)^2 \\ & + \frac{1}{2}\left(\frac{v}{2}\right)^2 \Bigg(-gA_\mu^3(x) + g' B_\mu^0(x) \\ & \qquad + g' \sum_{n=1}^{\infty} \frac{\chi^{(n)}(\pi R)}{\chi^{(0)}(\pi R)} B_\mu^n(x)\Bigg)^2, \end{aligned} \tag{A.12}$$

where $g' = g_5\chi^{(0)}(\pi R)$. By considering the new basis and using the mixing angle θ_W, we rewrite the above Lagrangian in terms of photon and Z fields as

$$\begin{aligned} L_\mathrm{m} = & \sum_{n=1}^{\infty} \frac{1}{2} m_n^2 \left(B_\mu^{(n)}(x)\right)^2 \\ & + \frac{1}{2}\left(m_Z Z_\mu(x) - \frac{g'v}{2} \sum_{n=1}^{\infty} \frac{\chi^{(n)}(\pi R)}{\chi^{(0)}(\pi R)} B_\mu^n(x)\right)^2. \end{aligned} \tag{A.13}$$

Here the photon field is massless, as it should, and there exists mixing among Z boson and B field KK modes. With the assumption that $m_Z \ll m_n$, the diagonalization of the mass matrix results in a shift in the mass of the Z boson as

$$m_Z^\mathrm{phys} = m_Z \sqrt{1 - \left(\frac{g'v}{2}\right)^2 \sum_{n=1}^{\infty} \left(\frac{1}{m_n}\frac{\chi^{(n)}(\pi R)}{\chi^{(0)}(\pi R)}\right)^2}\,, \tag{A.14}$$

and the other physical masses coming from the mixing can be approximated to m_n (see (A.11)). On the other hand, the coupling of a zero mode (KK mode) physical Z boson to the fermions reads $g' = g_5\chi^{(0)}$ ($g'^{(n)} \sim g_5\chi^{(n)}$). Here $g'^{(n)} \sim g'\sqrt{\frac{1-\mathrm{e}^{-2\alpha\pi k R}}{\alpha}}$ and, for positive α, $g'^{(n)} \sim g'\sqrt{\frac{1}{\alpha}}$. This is the case that the photon and Z bosons are effectively localized around the visible brane.

Appendix B: Explicit expressions

The explicit expressions for the functions f_1, f_2, f_3 and f_4 appearing in (5) read

$$\begin{aligned} f_1 = & \frac{g}{64\pi^2\cos\theta_\mathrm{W}} \int_0^1 \mathrm{d}x \frac{1}{m_{l_2}^2 - m_{l_1}^2} \Bigg\{ c_V (m_{l_2} + m_{l_1}) \\ & \times \Bigg(\left(-m_i\eta_i^+ + m_{l_1}(-1+x)\eta_i^V\right) \ln \frac{L_{1,h^0}^\mathrm{self}}{\mu^2} \\ & + \left(m_i\eta_i^+ - m_{l_2}(-1+x)\eta_i^V\right) \ln \frac{L_{2,h^0}^\mathrm{self}}{\mu^2} \\ & + \left(m_i\eta_i^+ + m_{l_1}(-1+x)\eta_i^V\right) \ln \frac{L_{1,A^0}^\mathrm{self}}{\mu^2} \\ & - \left(m_i\eta_i^+ + m_{l_2}(-1+x)\eta_i^V\right) \ln \frac{L_{2,A^0}^\mathrm{self}}{\mu^2} \Bigg) \\ & + c_A (m_{l_2} - m_{l_1}) \\ & \times \Bigg(\left(-m_i\eta_i^- + m_{l_1}(-1+x)\eta_i^A\right) \ln \frac{L_{1,h^0}^\mathrm{self}}{\mu^2} \\ & + \left(m_i\eta_i^- + m_{l_2}(-1+x)\eta_i^A\right) \ln \frac{L_{2,h^0}^\mathrm{self}}{\mu^2} \\ & + \left(m_i\eta_i^- + m_{l_1}(-1+x)\eta_i^A\right) \ln \frac{L_{1,A^0}^\mathrm{self}}{\mu^2} \\ & + \left(-m_i\eta_i^- + m_{l_2}(-1+x)\eta_i^A\right) \ln \frac{L_{2,A^0}^\mathrm{self}}{\mu^2} \Bigg) \Bigg\} \\ & - \frac{g}{64\pi^2\cos\theta_\mathrm{W}} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \\ & \times \Bigg\{ m_i^2 \left(c_A\eta_i^A - c_V\eta_i^V\right) \left(\frac{1}{L_{A^0}^\mathrm{ver}} + \frac{1}{L_{h^0}^\mathrm{ver}}\right) \\ & - (1-x-y) m_i \Bigg(c_A (m_{l_2} - m_{l_1}) \eta_i^- \left(\frac{1}{L_{h^0}^\mathrm{ver}} - \frac{1}{L_{A^0}^\mathrm{ver}}\right) \\ & + c_V (m_{l_2} + m_{l_1}) \eta_i^+ \left(\frac{1}{L_{h^0}^\mathrm{ver}} + \frac{1}{L_{A^0}^\mathrm{ver}}\right) \Bigg) \\ & - \left(c_A\eta_i^A + c_V\eta_i^V\right) \Bigg(-2 + \left(k^2 xy + m_{l_1} m_{l_2} (-1+x+y)^2\right) \\ & \times \left(\frac{1}{L_{h^0}^\mathrm{ver}} + \frac{1}{L_{A^0}^\mathrm{ver}}\right) - \ln \frac{L_{h^0}^\mathrm{ver}}{\mu^2} \frac{L_{A^0}^\mathrm{ver}}{\mu^2} \Bigg) \\ & - (m_{l_2} + m_{l_1})(1-x-y) \\ & \times \Bigg(\frac{\eta_i^A (x m_{l_1} + y m_{l_2}) + m_i \eta_i^-}{2 L_{A^0 h^0}^\mathrm{ver}} \\ & + \frac{\eta_i^A (x m_{l_1} + y m_{l_2}) - m_i \eta_i^-}{2 L_{h^0 A^0}^\mathrm{ver}} \Bigg) \\ & + \frac{1}{2}\eta_i^A \ln \frac{L_{A^0 h^0}^\mathrm{ver}}{\mu^2} \frac{L_{h^0 A^0}^\mathrm{ver}}{\mu^2} \Bigg\}, \end{aligned}$$

$$\begin{aligned} f_2 = & \frac{g}{64\pi^2\cos\theta_\mathrm{W}} \int_0^1 \mathrm{d}x \frac{1}{m_{l_2}^2 - m_{l_1}^2} \Bigg\{ c_V (m_{l_2} - m_{l_1}) \\ & \times \Bigg(\left(m_i\eta_i^- + m_{l_1}(-1+x)\eta_i^A\right) \ln \frac{L_{1,A^0}^\mathrm{self}}{\mu^2} \\ & + \left(-m_i\eta_i^- + m_{l_2}(-1+x)\eta_i^A\right) \ln \frac{L_{2,A^0}^\mathrm{self}}{\mu^2} \\ & + \left(-m_i\eta_i^- + m_{l_1}(-1+x)\eta_i^A\right) \ln \frac{L_{1,h^0}^\mathrm{self}}{\mu^2} \end{aligned}$$

$$
\begin{aligned}
&+ \left(m_i \eta_i^- + m_{l_2}(-1+x)\eta_i^A\right) \ln \frac{L^{\text{self}}_{2,h^0}}{\mu^2}\Bigg) \\
&+ c_A (m_{l_2} + m_{l_1}) \\
&\times \Bigg(\left(m_i \eta_i^+ + m_{l_1}(-1+x)\eta_i^V\right) \ln \frac{L^{\text{self}}_{1,A^0}}{\mu^2} \\
&- \left(m_i \eta_i^+ + m_{l_2}(-1+x)\eta_i^V\right) \ln \frac{L^{\text{self}}_{2,A^0}}{\mu^2} \\
&+ \left(-m_i \eta_i^+ + m_{l_1}(-1+x)\eta_i^V\right) \ln \frac{L^{\text{self}}_{1,h^0}}{\mu^2} \\
&+ \left(m_i \eta_i^+ - m_{l_2}(-1+x)\eta_i^V\right) \ln \frac{L^{\text{self}}_{2,h^0}}{\mu^2}\Bigg) \Bigg\} \\
&+ \frac{g}{64\pi^2 \cos\theta_{\mathrm{W}}} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \\
&\times \Bigg\{ m_i^2 \left(c_V \eta_i^A - c_A \eta_i^V\right) \left(\frac{1}{L^{\text{ver}}_{A^0}} + \frac{1}{L^{\text{ver}}_{h^0}} \right) \\
&- m_i (1-x-y) \big(c_V (m_{l_2} - m_{l_1}) \eta_i^- \\
&+ c_A (m_{l_2} + m_{l_1}) \eta_i^+ \big) \left(\frac{1}{L^{\text{ver}}_{h^0}} - \frac{1}{L^{\text{ver}}_{A^0}} \right) \\
&+ \left(c_V \eta_i^A + c_A \eta_i^V\right) \Bigg(-2 + \left(k^2 xy - m_{l_1} m_{l_2} (-1+x+y)^2\right) \\
&\times \left(\frac{1}{L^{\text{ver}}_{h^0}} + \frac{1}{L^{\text{ver}}_{A^0}} \right) - \ln \frac{L^{\text{ver}}_{h^0}}{\mu^2} \frac{L^{\text{ver}}_{A^0}}{\mu^2} \Bigg) \\
&- (m_{l_2} - m_{l_1})(1-x-y) \\
&\times \Bigg(\frac{\eta_i^V (x m_{l_1} - y m_{l_2}) + m_i \eta_i^+}{2 L^{\text{ver}}_{A^0 h^0}} \\
&+ \frac{\eta_i^V (x m_{l_1} - y m_{l_2}) - m_i \eta_i^+}{2 L^{\text{ver}}_{h^0 A^0}} \Bigg) \\
&- \frac{1}{2} \eta_i^V \ln \frac{L^{\text{ver}}_{A^0 h^0}}{\mu^2} \frac{L^{\text{ver}}_{h^0 A^0}}{\mu^2} \Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
f_3 = &-\mathrm{i} \frac{g}{64\pi^2 \cos\theta_{\mathrm{W}}} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \\
&\times \Big\{ \big((1-x-y)\left(c_V \eta_i^V + c_A \eta_i^A\right)(x m_{l_1} + y m_{l_2}) \\
&+ m_i \left(c_A (x-y) \eta_i^- + c_V \eta_i^+ (x+y) \right) \big) \frac{1}{L^{\text{ver}}_{h^0}} \\
&+ \big((1-x-y)\left(c_V \eta_i^V + c_A \eta_i^A\right)(x m_{l_1} + y m_{l_2}) \\
&- m_i \left(c_A (x-y) \eta_i^- + c_V \eta_i^+ (x+y) \right) \big) \frac{1}{L^{\text{ver}}_{A^0}} \\
&- (1-x-y) \Bigg(\frac{\eta_i^A (x m_{l_1} + y m_{l_2})}{2} \left(\frac{1}{L^{\text{ver}}_{A^0 h^0}} + \frac{1}{L^{\text{ver}}_{h^0 A^0}} \right) \\
&+ \frac{m_i \eta_i^-}{2} \left(\frac{1}{L^{\text{ver}}_{h^0 A^0}} - \frac{1}{L^{\text{ver}}_{A^0 h^0}} \right) \Bigg) \Big\},
\end{aligned}
$$

$$
\begin{aligned}
f_4 = &-\mathrm{i} \frac{g}{64\pi^2 \cos\theta_{\mathrm{W}}} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \\
&\times \Big\{ \big((1-x-y)\left(-\left(c_V \eta_i^A + c_A \eta_i^V\right)(x m_{l_1} - y m_{l_2})\right) \\
&- m_i \left(c_A (x-y) \eta_i^+ + c_V \eta_i^- (x+y) \right) \big) \frac{1}{L^{\text{ver}}_{h^0}} \\
&+ \big((1-x-y)\left(-\left(c_V \eta_i^A + c_A \eta_i^V\right)(x m_{l_1} - y m_{l_2})\right) \\
&+ m_i \left(c_A (x-y) \eta_i^+ + c_V \eta_i^- (x+y) \right) \big) \frac{1}{L^{\text{ver}}_{A^0}} \\
&+ (1-x-y) \Bigg(\frac{\eta_i^V}{2} (m_{l_1} x - m_{l_2} y) \left(\frac{1}{L^{\text{ver}}_{A^0 h^0}} + \frac{1}{L^{\text{ver}}_{h^0 A^0}} \right) \\
&+ \frac{m_i \eta_i^+}{2} \left(\frac{1}{L^{\text{ver}}_{A^0 h^0}} - \frac{1}{L^{\text{ver}}_{h^0 A^0}} \right) \Bigg) \Big\}, \qquad \text{(B.1)}
\end{aligned}
$$

where

$$
\begin{aligned}
L^{\text{self}}_{1,h^0} &= m^2_{h^0}(1-x) + \left(m_i^2 - m_{l_1}^2 (1-x)\right) x, \\
L^{\text{self}}_{1,A^0} &= L^{\text{self}}_{1,h^0}(m_{h^0} \to m_{A^0}), \\
L^{\text{self}}_{2,h^0} &= L^{\text{self}}_{1,h^0}(m_{l_1} \to m_{l_2}), \\
L^{\text{self}}_{2,A^0} &= L^{\text{self}}_{1,A^0}(m_{l_1} \to m_{l_2}), \\
L^{\text{ver}}_{h^0} &= m^2_{h^0}(1-x-y) + m_i^2 (x+y) - k^2 xy, \\
L^{\text{ver}}_{h^0 A^0} &= m^2_{A^0} x + m_i^2 (1-x-y) + \left(m^2_{h^0} - k^2 x\right) y, \\
L^{\text{ver}}_{A^0} &= L^{\text{ver}}_{h^0}(m_{h^0} \to m_{A^0}), \\
L^{\text{ver}}_{A^0 h^0} &= L^{\text{ver}}_{h^0 A^0}(m_{h^0} \to m_{A^0}), \qquad \text{(B.2)}
\end{aligned}
$$

and

$$
\begin{aligned}
\eta_i^V &= \xi^E_{\mathrm{N},l_1 i} \xi^{E*}_{\mathrm{N},i l_2} + \xi^{E*}_{\mathrm{N},i l_1} \xi^E_{\mathrm{N},l_2 i}, \\
\eta_i^A &= \xi^E_{\mathrm{N},l_1 i} \xi^{E*}_{\mathrm{N},i l_2} - \xi^{E*}_{\mathrm{N},i l_1} \xi^E_{\mathrm{N},l_2 i}, \\
\eta_i^+ &= \xi^{E*}_{\mathrm{N},i l_1} \xi^{E*}_{\mathrm{N},i l_2} + \xi^E_{\mathrm{N},l_1 i} \xi^E_{\mathrm{N},l_2 i}, \\
\eta_i^- &= \xi^{E*}_{\mathrm{N},i l_1} \xi^{E*}_{\mathrm{N},i l_2} - \xi^E_{\mathrm{N},l_1 i} \xi^E_{\mathrm{N},l_2 i}. \qquad \text{(B.3)}
\end{aligned}
$$

The parameters c_V and c_A are $c_A = -\frac{1}{4}$ and $c_V = \frac{1}{4} - \sin^2\theta_{\mathrm{W}}$. In (B.3) the flavor changing couplings $\xi^E_{\mathrm{N},ji}$ represent the effective interaction between the internal lepton i, $(i = e, \mu, \tau)$ and outgoing (incoming) $j = l_1$ $(j = l_2)$ one. Here we take only the τ lepton in the internal line and we neglect all the Yukawa couplings except $\xi^E_{\mathrm{N},\tau\tau}$ and $\xi^E_{\mathrm{N},\tau\mu}$ in the loop contributions (see Sect. 3). The Yukawa couplings $\xi^E_{\mathrm{N},ji}$ are complex in general; however, in the present work, we take them real.

References

1. MEGA Collaboration, M.L. Brooks et al., Phys. Rev. Lett. **83**, 1521 (1999)
2. CLEO Collaboration, S. Ahmed et al., Phys. Rev. D **61**, 071 101 (2000)
3. E. Iltan, H. Sundu, J. Phys. G **29**, 1373 (2003)
4. E. Iltan, Mod. Phys. Lett. A **20**, 1845 (2005)
5. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999)
6. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999)
7. W.D. Goldberger, M.B. Wise, Phys. Rev. Lett. **83**, 4922 (1999)
8. A. Pomarol, Phys. Lett. B **486**, 153 (2000)
9. H. Davoudias, J.L. Hewett, T.G. Rizzo, Phys. Lett. B **473**, 43 (2000)
10. T. Gherghetta, A. Pomarol, Nucl. Phys. B **586**, 141 (2000)
11. Y. Grossman, M. Neubert, Phys. Lett. B **474**, 361 (2000)

12. I.I. Kogan, S. Mouslopolous, A. Papazoglou, G.G. Ross, Nucl. Phys. B **615**, 191 (2001)
13. T. Gherghetta, A. Pomarol, Nucl. Phys. B **602**, 3 (2001)
14. S.J. Huber, Q. Shafi, Phys. Rev. D **63**, 045 010 (2001)
15. K. Ghoroku, A. Nakamura, Phys. Rev. D **65**, 084 017 (2002)
16. B. Batell, T. Gherghetta, Phys. Rev. D **73**, 045 016 (2006)
17. E. Iltan, H. Sundu, Acta Phys. Slovaca **53**, 17 (2003)